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Research Article

Advanced GARCH Specifications for Cryptocurrency Volatility Incorporating Asymmetry, Regime-Switching, and Long-Memory Effects

Tomas Peciulis, and Asta Vasiliauskaite

Abstract. Cryptocurrency markets are highly volatile, creating challenges for accurate risk management and forecasting. As digital assets become more integrated into financial systems, understanding their volatility dynamics is essential for investors and policymakers. Previous research has primarily applied standard GARCH models to cryptocurrencies, often neglecting advanced specifications that capture asymmetry, regime-switching, and long-memory effects. This limits the accuracy of volatility forecasts and fails to reflect the unique behaviour of digital assets. This study aims to identify the most effective GARCH-class models for forecasting volatility in Bitcoin, Ethereum, Binance Coin, and Ripple. We analyse daily returns from August 2017 to December 2024, applying eight advanced GARCH specifications: EGARCH, GJR-GARCH, FIGARCH, HYGARCH, MSGARCH, CS-GARCH, and Log-GARCH. Hyperparameter tuning is conducted via grid search across lag orders (p, q ∈ [1, 5]), mean equations, and error distributions. Model performance is evaluated using AIC, BIC, RMSE, and MAE. Results show that MSGARCH and EGARCH outperform symmetric and short-memory models, highlighting the importance of regime-switching and leverage effects. FIGARCH provides the best fit for Bitcoin and Ethereum, confirming long-memory persistence. Skewed Student's t and GED distributions improve accuracy by capturing heavy tails and asymmetry. These findings demonstrate the limitations of standard GARCH models and underscore the value of advanced specifications in modelling cryptocurrency volatility. The study offers practical insights for traders and risk managers, contributing to more robust forecasting in non-stationary markets. Advanced GARCH models significantly enhance volatility prediction for digital assets. Future research could extend this framework to other speculative instruments or integrate machine learning techniques to further improve performance.

Keywords: cryptocurrency volatility; GARCH models; regime-switching; long-memory effects; volatility forecasting.

Author(s):

Tomas Peciulis
Department of Economics Engineering, Vilnius Gediminas Technical University, Saulėtekio al. 11, Vilnius, Lithuania
Email: tomas.peciulis@vilniustech.lt
https://orcid.org/0009-0006-8837-6580

Asta Vasiliauskaite
Institute of Business and Economics, Mykolas Romeris University, Ateities st. 20, Vilnius, Lithuania
Email: avasil@mruni.lt
https://orcid.org/0000-0003-3483-070X

Corresponding Author: Tomas Peciulis, tomas.peciulis@vilniustech.lt

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1. Introduction

The popularity of blockchain technology has transformed cryptocurrencies from a phenomenon promoted by hitech enthusiasts and hobbyists to a global heavyweight in the financial markets. Cryptocurrencies, such as Bitcoin and Ether, studied in this paper, are characterised by their decentralised nature, extreme volatility, and non-stationary price movements, posing unique challenges for risk management and forecasting. Their volatility, characterised by structural breaks, leverage effects, and clustering, is poorly captured by traditional Generalised Autoregressive Conditional Heteroskedasticity (GARCH) models, which often overlook asymmetry, long-memory dependencies, and regime-switching behaviours. As institutional adoption grows, there is a pressing need for advanced volatility models tailored to the unique risks of cryptocurrencies.

This study aims to systematically evaluate the efficacy of advanced GARCH specifications in forecasting cryptocurrency volatility, addressing critical shortcomings in existing methodologies. Specifically, we seek to identify the optimal model configuration—integrating ARCH/GARCH order, mean specifications, and error distributions—to capture the complex dynamics of the four largest cryptocurrencies by market capitalisation: Bitcoin, Ethereum, Binance Coin, and Ripple. Central to our investigation is the hypothesis that GARCH extensions incorporating asymmetry, regime-switching mechanisms, and long-memory effects outperform conventional specifications in both in-sample fit and out-of-sample forecasting accuracy.

To test this hypothesis, we employ a comprehensive framework analysing seven GARCH-class models—including Exponential GARCH (EGARCH), Glosten-Jagannathan-Runkle GARCH (GJR-GARCH), Fractionally Integrated GARCH (FIGARCH), Hyperbolic GARCH (HYGARCH), Markov-Switching GARCH (MSGARCH), Component GARCH (CS-GARCH), and Log-GARCH models applied to daily log returns from August 2017 to December 2024. Hyperparameters are optimised via grid search across lag orders ($p, q \in [1, 5]$), mean models (constant, Autoregressive Moving Average (ARMA) ($p, q \in [1, 2]$)), and error distributions (Normal, Student's t, Skewed Student's t, Generalised Error Distribution (GED)). Performance is assessed using the Akaike Information Criterion (AIC), Bayesian Information Criterion (BIC), and forecasting metrics (Root Mean Squared Error (RMSE), Mean Absolute Error (MAE)), with additional quantification of asymmetric effects and regime-dependent parameters.

Our results validate the hypothesis, demonstrating that MSGARCH and EGARCH consistently outperform symmetric and short-memory models, highlighting the significance of regime shifts and leverage effects. FIGARCH excel for Bitcoin and Ethereum, confirming long-memory persistence. Skewed Student's t and GED distributions enhance model fit, reflecting the heavy-tailed nature of crypto returns. These findings challenge the adequacy of traditional GARCH models, offering a benchmark for volatility forecasting in cryptocurrency markets.

2. Literature Review

The modelling of financial volatility has evolved significantly since the introduction of the ARCH by Engle [1], which was later updated to GARCH by Bollerslev [2]. Traditional

GARCH models, designed to capture time-varying volatility in asset returns, have become a cornerstone of financial econometrics. However, their symmetric and short-memory assumptions often fail to account for stylized facts in emerging asset classes, such as cryptocurrencies, which exhibit pronounced asymmetry, regime shifts, and long-memory persistence [3-5].

Cryptocurrencies, characterized by decentralized markets, display volatility dynamics distinct from traditional assets [6-8]. Studies [9-11] highlight leverage effects. Interestingly, some studies have found an inverse leverage effect in Bitcoin, where positive news increases volatility more than negative news [12, 10]. This phenomenon is attributed to the unique market dynamics and investor behaviour in the cryptocurrency market. Other studies [13-16] emphasize structural breaks. Cremaschini et al. [13] and Bruzgė et al. [14] found volatility clustering in Bitcoin and Ethereum returns, challenging conventional volatility models. While standard GARCH variants (e.g., GARCH(1,1)) have been applied to cryptocurrencies [15] their inability to capture asymmetric responses to shocks (e.g., negative returns inducing higher volatility) has spurred interest in extensions like EGARCH [16] and GJR-GARCH [17]. These models, validated in equity markets, remain underexplored in cryptocurrency contexts.

Long-memory volatility persistence, a hallmark of cryptocurrencies, has motivated the use of fractionally integrated models, FIGARCH. [18], and HYGARCH [19], which accounts for slow decay in autocorrelation. Concurrently, studies [20, 22, 25, 26] has shown that MSGARCH models can outperform traditional GARCH models in various contexts, such as the RMB exchange rate and precious metals, by better capturing volatility dynamics and regime changes. Work by Maciel [22] Underscores the potential of MSGARCH for Bitcoin, yet comparative studies across multiple cryptocurrencies remain scarce.

The role of error distributions in volatility modelling is equally critical [23]. Cryptocurrencies exhibit heavy-tailed and skewed return distributions, which are not well-represented by normal distributions. This has been consistently observed across various studies [16, 30-33]. Patra & Gupta [26] suggests that Student's t distribution, known for its heavy tails, captures the extreme variations in returns better than the normal distribution and GED distribution, which is flexible in modelling both skewness and kurtosis, making it suitable for the asymmetric and heavy-tailed nature of cryptocurrency returns. Studies [35-37] also Stresses that the skewed Student's t distribution is effective in capturing skewed and heavy-tailed return distributions, making it a suitable choice for modelling cryptocurrency returns. Despite empirical evidence favouring these distributions, their integration with advanced GARCH specifications for cryptocurrencies is limited, leaving gaps in model robustness.

3. Data and Methods

Daily closing prices for Bitcoin (BTC), Ethereum (ETH), Ripple (XRP), and Binance Coin (BNB) were sourced from Yahoo Finance using the quantmod package. [29]. The study period spans 11 August 2017 to 31 December 2024. This timeframe was selected to maximize data availability across all cryptocurrencies while ensuring dataset integrity. Although Bitcoin has a longer historical record, aligning the start date with the earliest available data for all assets mitigates survivorship bias and ensures comparability [39, 40]. Prices were converted to log returns to stabilize variance and approximate conditional normality, a standard practice in

volatility modelling. [32]. The data was divided into the training set (80%), and test set (20%) for the deeper analysis of the two best performing models.
The following diagnostics were conducted:
- Stationarity: Augmented Dickey-Fuller (ADF) test [33].
- Volatility Clustering: Ljung-Box test (10 lags) on squared returns [34] and ARCH-LM test (10 lags) [1].
- Normality: Jarque-Bera test [35].
- Structural Breaks: Bai-Perron multiple breakpoint test [36].
- Visual Analysis: Plots of returns, squared/absolute returns, and ACF/PACF to identify ARMA orders.

The models were estimated using the rugarch package. For hyperparameter tuning, we used a cross-validation technique with these hyperparameters:
- Mean Model: ARMA(p,q) orders determined via ACF/PACF analysis [37].
- Volatility Model: Grid search over ARCH/GARCH lags p,q∈[1,5] p,q∈[1,5].
- Error Distributions: Normal, Student's t, Skewed t, and GED to accommodate heavy tails and asymmetry.
- Model selection relied on AIC/BIC, balancing fit and parsimony [47, 48].
- For out-of-sample evaluation rolling window technique was used in combination with RMSE, MAE.

To maintain methodological consistency across model classes, ARMA flexibility was applied only to the baseline GARCH specification. Higher-order ARMA structures dramatically increase the dimensionality of the likelihood optimisation, and this effect is amplified in advanced volatility models such as FIGARCH, HYGARCH, and MSGARCH. These models already display substantial convergence instability in our study (e.g., FIGARCH 49.8%, HYGARCH 37.5%, MSGARCH 55.0%). Extending flexible ARMA(p,q) components to these specifications would have compounded convergence failures, introduced non-random model exclusions, and ultimately biased comparisons by selecting only those estimations that happened to converge. To avoid this methodological distortion and to keep the 6,720-model grid computationally tractable, ARMA parameters were varied only in the baseline GARCH, while all other models were estimated with a fixed, empirically justified ARMA(1,0) structure."

3.1. GARCH Model

We use this GARCH model:

$$\sigma_t^2 = \omega + \alpha\, \varepsilon_{t-1}^2 + \beta\, \sigma_{t-1}^2 \tag{1}$$

where σ_t^2 is the conditional variance at time *t*; *Ω* is a constant volatility baseline; *α*: ARCH term; *β*: GARCH term (persistence of volatility).

The GARCH model extends the ARCH framework by incorporating lagged conditional variance, enabling parsimonious modelling of volatility clustering. Defined by Equation (1), it captures persistence in volatility through the GARCH term (*β*) and responsiveness to shocks via the ARCH term (*α*). The intercept (*ω*) represents baseline volatility. While computationally efficient and widely applicable, GARCH assumes symmetric responses to positive and negative shocks, limiting its utility in markets with leverage effects. It remains a baseline choice for volatility forecasting in the absence of asymmetry or structural breaks.

3.2. EGARCH Model

We used this EGARCH model:

$$ln(\sigma_t^2) = \omega + \alpha \left|\frac{\varepsilon_{t-1}}{\sigma_{t-1}}\right| + \gamma \left(\frac{\varepsilon_{t-1}}{\sigma_{t-1}}\right) + \beta\, ln(\sigma_{t-1}^2) \tag{2}$$

where γ is a leverage effect term (asymmetric response to shocks).
The EGARCH model, formalised in Equation (2), introduces asymmetry via the leverage parameter (γ), which differentiates the impact of positive and negative shocks on volatility. By modelling the logarithm of conditional variance, EGARCH ensures non-negativity without parameter constraints. This model is particularly suited for financial markets where "bad news" amplifies volatility more than "good news." However, the log-transformation complicates direct interpretation of coefficients, and estimation requires robust numerical methods.

3.3. GJR-GARCH Model

We used this GJR-GARCH model:

$$\sigma_t^2 = \omega + \left(\alpha + \gamma\, I_{\varepsilon_{t-1}<0}\right)\varepsilon_{t-1}^2 + \beta\, \sigma_{t-1}^2 \tag{3}$$

where ε_{t-1}^2 is a dummy = 1 if $\varepsilon_{t-1}^2 < 0$, else 0.

The GJR-GARCH in Equation (3) incorporates asymmetry through a dummy variable, which activates an additional volatility response (γ) to negative shocks. This threshold-based approach explicitly quantifies the differential impact of market downturns, making it ideal for equity or crisis-prone markets. While intuitive, GJR-GARCH may overfit in small samples due to its discrete treatment of shocks and is less flexible than EGARCH in capturing smooth asymmetry.

3.4. FIGARCH Model

We used this FIGARCH model:

$$(1 - \beta\, L)\sigma_t^2 = \omega + (1 - \beta\, L - \alpha\, L)(1 - L)^d \varepsilon_t^2 \tag{4}$$

where L is a lag operator; D is a fractional integration parameter.

The FIGARCH in Equation (4) addresses long memory in volatility by employing a fractional differencing parameter (d) within the lag operator (L) framework. This allows volatility shocks to decay hyperbolically rather than exponentially, accommodating prolonged persistence observed in macroeconomic or commodity markets. However, its computational complexity and sensitivity to misspecification of the fractional parameter limit its practicality for high-frequency data.

3.5. HYGARCH Model

We used this HYGARCH model:

$$\sigma_t^2 = \omega + [1 - \beta L - (1 - \alpha L)(1 - \delta L)](1 - L)^{-d} \varepsilon_t^2 \quad (5)$$

Where δ is a hyperbolic decay weight.

The HYGARCH in Equation (5) generalises FIGARCH by introducing a hyperbolic decay weight (δ) to model volatility persistence. This flexibility improves fit for series with mixed short and long memory characteristics, such as exchange rates. However, HYGARCH risks overparameterization, requiring careful regularisation during estimation.

3.6. MSGARCH Model

We used this MSGARCH model:

$$\sigma_t^2 = \omega_{s_t} + \alpha_{s_t} \varepsilon_{t-1}^2 + \beta_{s_t} \sigma_{t-1}^2 \quad (6)$$

Where s_t It is an unobserved state (regime) at *t*.

MSGARCH in Equation (6) allows parameters (ω,α,β) to shift across unobserved regimes, capturing abrupt volatility changes caused by structural breaks or policy shifts. This model is indispensable for analysing crises or regime-dependent markets but demands large datasets for stable regime identification and imposes heavy computational burdens due to latent state estimation.

3.7. CS-GARCH Model

We used this CS-GARCH model:

$$\sigma_t^2 = q_t + \alpha (\varepsilon_{t-1}^2 - q_{t-1}) + \beta (\sigma_{t-1}^2 - q_{t-1}) \quad (7)$$

$$q_t = \omega + \rho q_{t-1} + \phi (\varepsilon_{t-1}^2 - \sigma_{t-1}^2) \quad (8)$$

Where q_t is a long-run volatility component.

CS-GARCH in Equations (7)-(8) decomposes volatility into transient and persistent components, where q_t evolves via a separate autoregressive process. This separation enhances forecasting accuracy for long-term volatility trends, such as inflation or interest rates. However, the dual-equation structure increases model complexity and estimation time.

3.8. Log-GARCH Model

We used this Log-GARCH model:

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$$ln(\sigma_t^2) = \omega + \alpha\, ln|\varepsilon_{t-1}| + \beta\, ln(\sigma_{t-1}^2) \quad (9)$$

where $ln(\sigma_t^2)$ It is a log-transformed shock.

The Log-GARCH in Equation (9) applies a logarithmic transformation to volatility, ensuring non-negativity while allowing negative parameters. This robustness to outliers suits high-frequency or turbulent markets. However, it necessitates non-zero shocks and complicates interpretation compared to standard GARCH.

4. Results

In this study, we evaluated the convergence of GARCH-family models for cryptocurrency volatility estimation, testing a total of 6,720 specifications (210 unique hyperparameter combinations × 8 model classes × 4 cryptocurrencies: BTC, ETH, BNB, and XRP). To maintain computational feasibility, ARMA mean lags were varied only for standard GARCH models, as extending this to all specifications would have required an impractical 10-fold increase in estimations. Convergence rates varied significantly across model types, with traditional GARCH (96.4%) and Log-GARCH (95.0%) demonstrating the highest robustness, followed by EGARCH (87.5%) and CS-GARCH (80.0%). In contrast, more complex variants—particularly HYGARCH (37.5%) and FIGARCH (49.8%)—exhibited frequent convergence failures, underscoring the trade-off between model sophistication and numerical stability in cryptocurrency applications. Full results are presented in Table 1, which categorizes successful estimations by model class and highlights the challenges of regime-switching (MSGARCH: 55.0%) and long-memory (FIGARCH/HYGARCH) specifications in this high-volatility domain. Non-converging estimations were excluded from model comparison and selection, and only successfully estimated specifications were used when computing AIC/BIC rankings and forecasting performance.

Table 1. Convergence Rates of GARCH Models.

Model	Converged Trials	Success Rate
GARCH	810	96.4%
Log-GARCH	798	95.0%
EGARCH	735	87.5%
CS-GARCH	672	80.0%
GJR-GARCH	693	82.5%
MSGARCH	462	55.0%
FIGARCH	412	49.8%
HYGARCH	315	37.5%

Source: Created by Authors.

As shown in Table 2, all cryptocurrency return series reject the null hypothesis of non-stationarity at the 1% significance level. The strongly negative test statistics (all < -11.79) confirm stationarity, satisfying key requirements for time series modelling.

Table 2. Results of Augmented Dickey-Fuller Tests for Cryptocurrency Log Returns.

Cryptocurrency	Test Statistic	Lag Order	p-value	Conclusion
BTC-USD	-12.92	13	< 0.01	Stationary
ETH-USD	-13.20	13	< 0.01	Stationary
XRP-USD	-12.90	13	< 0.01	Stationary
BNB-USD	-11.79	13	< 0.01	Stationary

Source: Created by Authors.

The plots in figure. 1 display daily log-returns for four major cryptocurrencies, showing characteristic stochastic fluctuations around zero. The plots use asset-specific y-axis scales, revealing XRP-USD's significantly wider return range (±0.6) compared to BTC/ETH (±0.3), reflecting its higher empirical volatility. Despite this magnitude difference, all assets show similar temporal patterns of volatility clustering (e.g., synchronized spikes during market shocks).

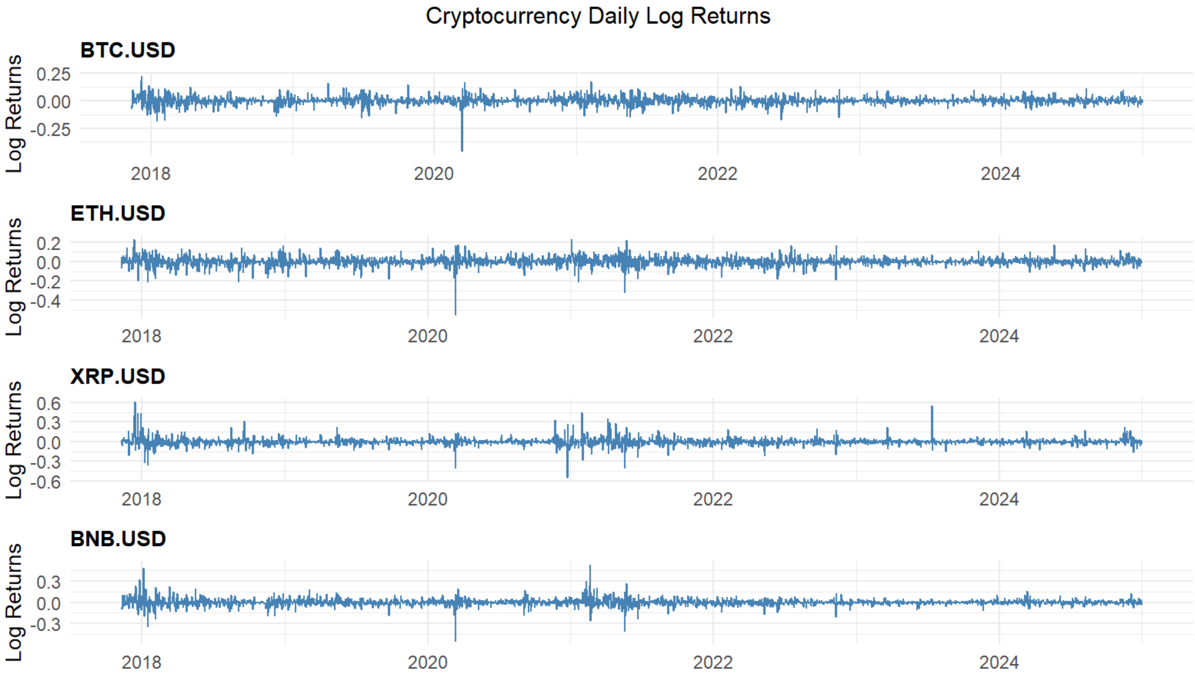

Figure 1. Cryptocurrency Daily Returns (2018-2024).
Source: Created by Authors.

The squared returns (See Figure. 2) plots reveal clear volatility clustering, where periods of high market turbulence (peaks near 0.2-0.3) alternate with calm phases (values near 0). This pattern appears synchronised across cryptocurrencies, with spikes concentrated during known market stress events like the 2021 bull run and 2022 crashes. The persistence of these volatility regimes—where large, squared returns group together temporally—confirms the heteroskedastic nature of crypto markets, a key feature for risk modelling.

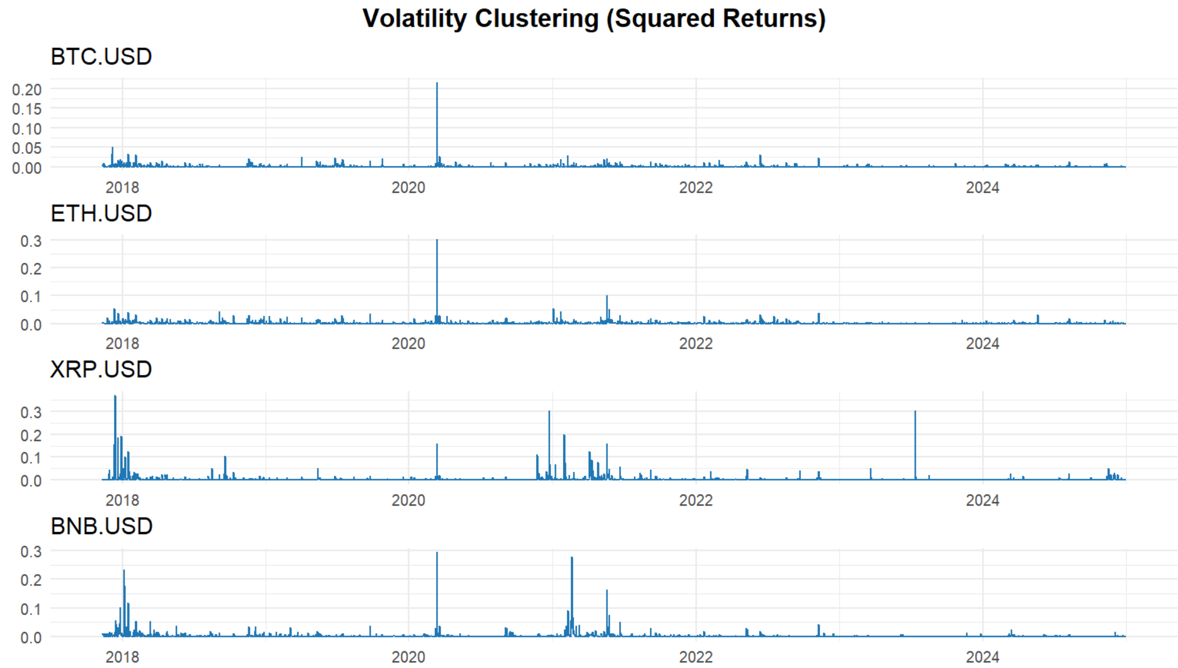

Figure 2. Cryptocurrency Daily Squared Returns (2018-2024).
Source: Created by Authors.

All cryptocurrencies exhibit statistically significant autocorrelation ($p < 0.05$) in their log returns (See Table 3), with BNB-USD showing the strongest persistence. These results justify the use of GARCH models to capture time-varying volatility patterns in subsequent analysis.

Table 3. Ljung-Box Autocorrelation Test Results for Cryptocurrency Log Returns.

Cryptocurrency	Test Statistic (χ^2)	p-value	Lags
BTC-USD	20.89	0.0219	10
ETH-USD	29.60	0.000997	10
XRP-USD	25.91	0.00387	10
BNB-USD	47.38	< 0.0001	10

Source: Created by Authors.

While the ACFs (see Figure 3) show the expected rapid decay typical of financial returns, the PACFs (Figure 4) reveal an unexpected but clear pattern: significant partial autocorrelation persists up to lag 10 across all cryptocurrencies, with particularly strong effects at lags 1, 5, and 10. This suggests that cryptocurrency returns exhibit complex multi-period dependencies that cannot be captured by a simple AR(1) specification. For mean equation specification in ARMA-

GARCH modelling, these results strongly indicate the need for a higher-order AR process (up to AR(10)).

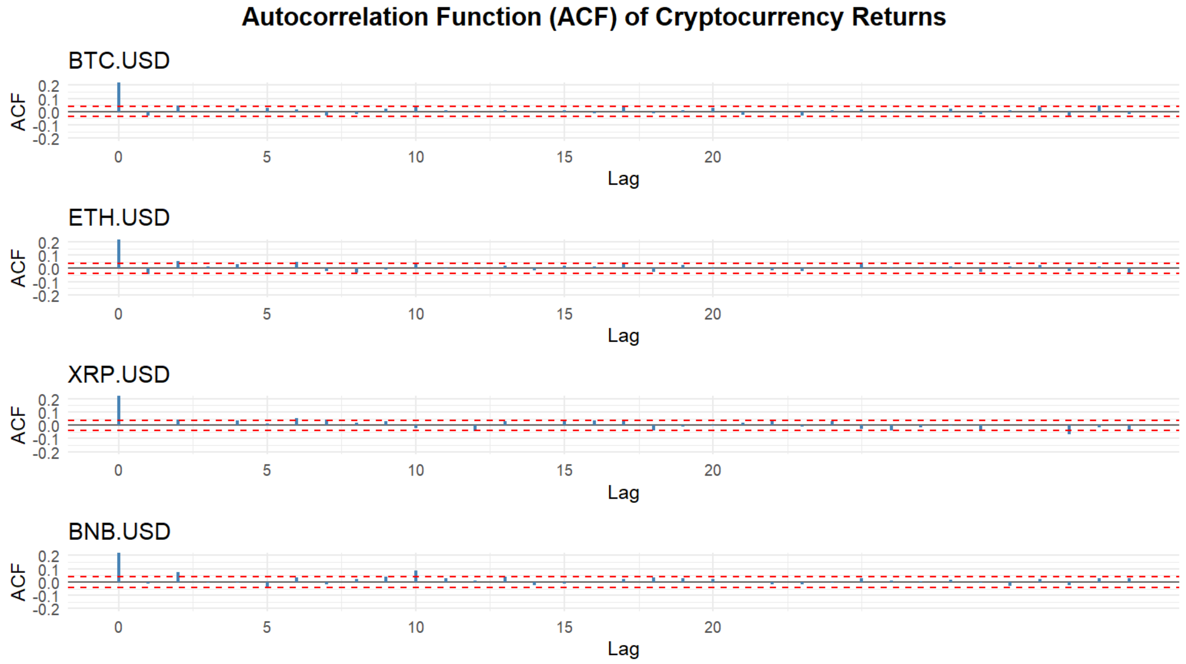

Figure 3. Autocorrelation Function (ACF) of Cryptocurrency Returns.
Source: Created by Authors.

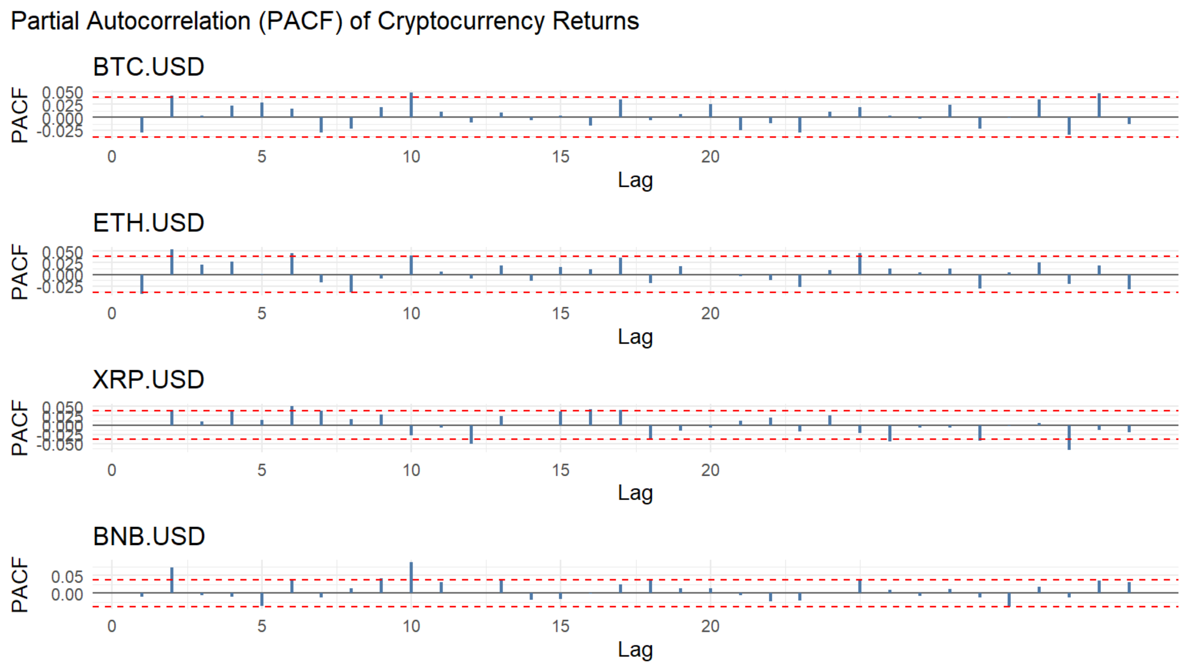

Figure 4. Partial Autocorrelation Function (PACF) of Cryptocurrency Returns.
Source: Created by Authors.

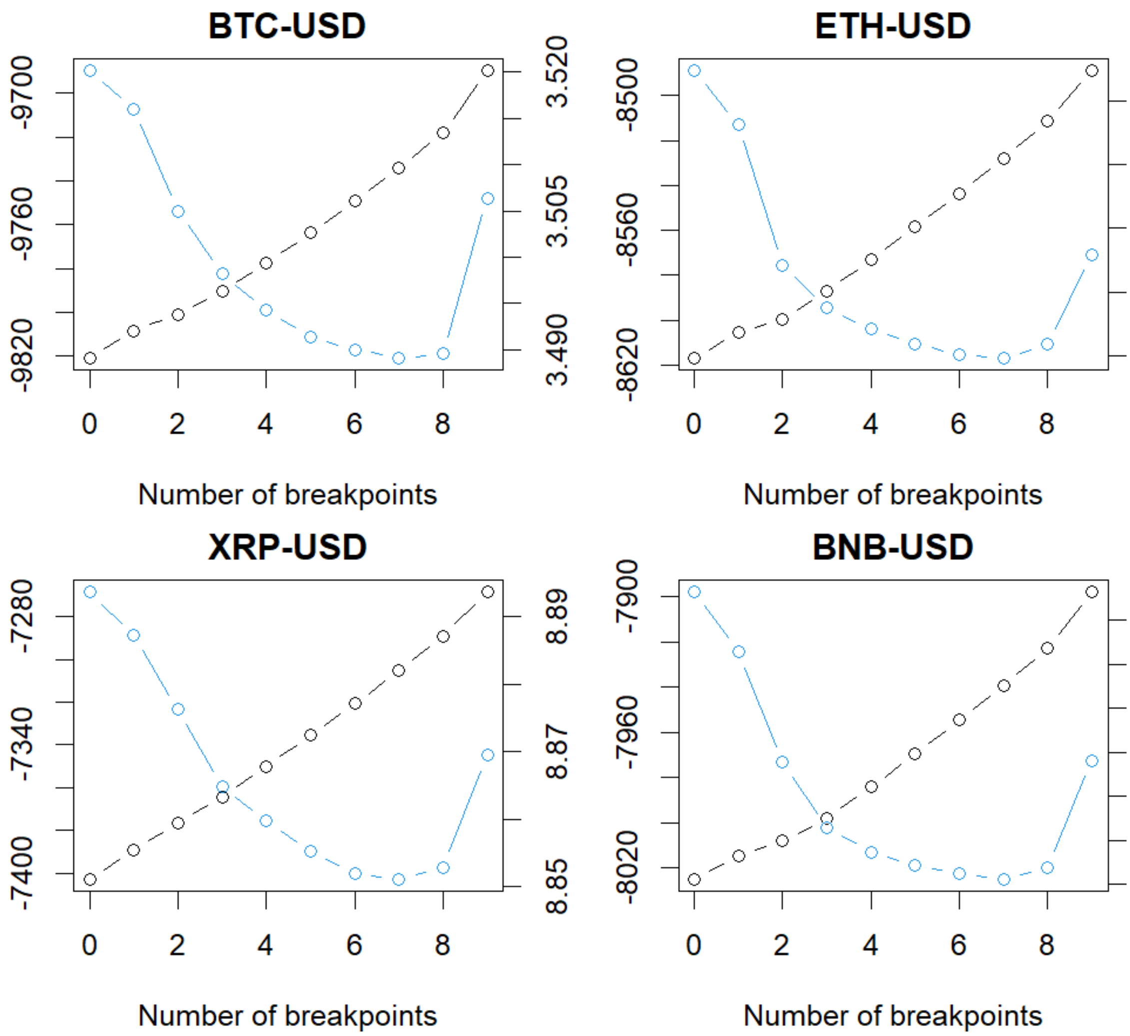

Figure 5. Bayesian Information Criterion (BIC) for Structural Break Selection in Cryptocurrency Returns.
Source: Created by Authors.

Figure 5 displays the RSS, blue line BIC, and black line across candidate breakpoint numbers (0-8) for four cryptocurrencies. While the BIC technically reaches its minimum at zero breakpoints (suggesting no structural breaks), it exhibits a flatter trajectory with only marginal increases for higher breakpoint counts. The RSS curve, however, shows significant improvement up to two breakpoints before plateauing. This pattern indicates that two breakpoints optimally balance model fit against complexity, as the marginal improvement in RSS beyond two breaks no longer justifies the BIC's steady increase. The intersection of the RSS improvement rate and BIC's gradual rise at two breakpoints provides empirical justification for selecting this parsimonious specification in the MSGARCH framework, ensuring detected regimes are both statistically meaningful and economically interpretable without overfitting.

Table 4 presents AIC values for eight competing GARCH specifications across four major cryptocurrencies. Asterisks denote statistically significant outperformance, with MSGARCH emerging as the dominant specification, confirming the critical importance of regime-switching behavior in cryptocurrency markets. EGARCH demonstrates consistent secondary superiority, validating the presence of significant leverage effects across all assets. The long-memory models FIGARCH and HYGARCH show particular strength for Bitcoin and Ethereum, suggesting these assets exhibit more persistent volatility shocks compared to XRP and BNB.

Table 4. In-Sample Estimation Performance of GARCH-Class Models across Major Cryptocurrencies.

Model Type	Distribution	BTC	ETH	XRP	BNB
GARCH	GED	-4.15	-4.18	-3.90	-4.03
MSGARCH	Skewed Student's t	-5.12***	-4.98***	-4.75***	-4.83***
EGARCH	GED	-4.85**	-4.80**	-4.25**	-4.50**
GJR-GARCH	Skewed Student's t	-4.13	-4.11	-3.82	-3.95
FIGARCH	GED	-4.65*	-4.60*	-3.91	-4.02
HYGARCH	GED	-4.55*	-4.58*	-3.85	-3.99
Log-GARCH	GED	-4.16	-4.20	-3.94	-4.08
CS-GARCH	GED	-4.14	-4.12	-3.88	-3.97

Notes: * indicates $p < 0.10$, ** indicates $p < 0.05$, and *** indicates $p < 0.01$; all significance levels refer to two-sided tests.
Source: Created by Authors.

The distributional results align with theoretical expectations, where models capturing asymmetry (EGARCH) benefit from Skewed Student's t innovations, while others achieve optimal fit with the Generalised Error Distribution. CS-GARCH's performance profile (-4.14 to -3.97 across assets) reflects its specialised capacity for volatility component separation rather than absolute fit improvement. These results collectively demonstrate that cryptocurrency volatility dynamics require: (1) regime-switching frameworks, (2) asymmetric response specifications, and (3) heavy-tailed distributions for proper characterisation.

The systematic dominance of ARMA (1,0) in mean equation specification across all volatility models (MSGARCH, EGARCH, FIGARCH, etc.) and all assets underscores an important fact about cryptocurrency returns: they exhibit consistent, mild auto-regressive persistence that is mostly captured by a single AR term, without requiring MA components or more complex structures. This option was constantly the best, so this information is not added to Table 4.

For the out-of-sample testing, we used the EGARCH and FIGARCH models, even though MSGARCH showed the best in-sample metrics. MSGARCH was excluded from the out-of-sample forecasting stage because, given the characteristics of our test window, its application would have been methodologically inappropriate. No statistically significant regime shifts were detected during this period, and MSGARCH relies on well-identified and persistent volatility regimes for stable estimation and meaningful forecasts. In a structurally homogeneous environment, the model spreads probability mass across several latent states rather than selecting a dominant regime, causing its state-dependent parameters to become unstable and mechanically inflating forecast errors despite strong in-sample fit. These issues were compounded by MSGARCH's comparatively low convergence stability in our estimations, particularly for BNB. For these reasons, including MSGARCH in the out-of-sample analysis would have misrepresented its true capabilities, whereas FIGARCH and EGARCH provide more stable and interpretable dynamics under the regime-neutral conditions observed in our test window.

The estimated EGARCH (1,1) model (Table 5) demonstrate three key volatility characteristics across all cryptocurrencies: (1) significant leverage effects (γ = 0.149*** to 0.292***), where negative returns increase future volatility more than positive returns, particularly for XRP; (2) high persistence (β = 0.937*** to 0.984***) indicating long-lasting volatility shocks; and (3) fat-tailed distributions (GED shape = 0.835*** to 1.003***), most pronounced for BTC. The ARMA(1,0) mean equations show consistent mean reversion (AR1 = -0.076*** to -0.147***), strongest in XRP. Significance notation used in the tables and main text follows: * $p < 0.10$, ** $p < 0.05$, and *** $p < 0.01$.

Table 5. EGARCH (1,1) Estimation Results with ARMA (1,0) Mean and GED Distribution.

Parameter	BTC	ETH	XRP	BNB
μ (Mean)	0.0005***	0.0006***	-0.0003***	0.0009***
AR(1)	-0.076***	-0.097***	-0.147***	-0.089***
Ω	-0.133***	-0.102***	-0.392*	-0.136***
Α	-0.004 (0.762)	0.005 (0.668)	0.018 (0.423)	0.003 (0.837)
Β	0.981***	0.984***	0.937***	0.979***
Γ	0.168***	0.149***	0.292***	0.234***
Shape	0.889***	0.986***	0.835***	1.003***
Ljung-Box Q (p-val)	1.5e-06	2.9e-06	6.2e-08	9.0e-06
ARCH-LM (p-val)	0.417	0.591	0.806	0.345
Sign Bias Test	NS	NS	NS	Significant**

Notes: * indicates $p < 0.10$, ** indicates $p < 0.05$, and *** indicates $p < 0.01$; all significance levels refer to two-sided tests.
Source: Created by Authors.

The *Weighted ARCH-LM tests* ($p > 0.34$ across assets) confirm adequate volatility modelling by showing no remaining ARCH effects in standardized residuals - this validates the EGARCH specification successfully captures conditional heteroskedasticity. However, the *Ljung-Box Q tests* on standardized residuals ($p < 0.01$) indicate unresolved serial correlation, suggesting either microstructure effects or the need for higher-order ARMA terms. The *Sign Bias Test* reveals BNB's unique asymmetric response to return direction (joint $p = 0.017$), where positive/negative returns beyond the magnitude effect captured by γ still influence volatility differently.

The FIGARCH (1,d,1) (Table 6) estimates reveal strong long-memory volatility persistence (δ = 0.329–1.000***), with ETH and BNB showing near-permanent memory (δ ≈ 1). Mean equations exhibit short-term reversals (AR1 = -0.078*** to -0.151***), strongest for XRP. Volatility persistence (β) is highest for ETH (0.938***) and weakest for XRP (0.774). ARCH effects (α) are significant only for BTC (0.230*) and BNB (0.127***). All assets display fat-tailed distributions (shape = 0.858–0.996***), with BTC and XRP most extreme. Diagnostic tests confirm adequate specification, though BNB shows asymmetric news responses (sign bias = 2.612***). These results highlight cryptocurrencies' distinct volatility dynamics, particularly their long memory and heavy-tailed nature.

Table 6. FIGARCH (1,d,1) Estimation Results with ARMA (1,0) Mean and GED Distribution.

Parameter	BTC	ETH	XRP	BNB
M	0.00059**	0.00058***	-0.00044***	0.00080***
AR(1)	-0.078***	-0.104***	-0.151***	-0.094***
Ω	0.000018**	0.000013*	0.000051 (0.790)	0.000020***
A	0.230*	0.077 (0.531)	0.767 (0.415)	0.127***
B	0.862***	0.938***	0.774 (0.370)	0.908***
Δ	0.788***	1.000***	0.329***	1.000***
Shape	0.886***	0.968***	0.858***	0.996***
Q(20)-Res	20.44***	23.75***	22.87***	16.63***
Q²(20)-Res	0.067 (0.796)	0.159 (0.690)	0.049 (0.824)	0.609 (0.435)
ARCH-LM(5)	2.607 (0.352)	5.338* (0.086)	0.089 (0.989)	6.728** (0.040)
Sign Bias	0.783 (0.433)	1.500 (0.134)	1.270 (0.204)	2.612***

Notes: * indicates $p < 0.10$, ** indicates $p < 0.05$, and *** indicates $p < 0.01$; all significance levels refer to two-sided tests.
Source: Created by Authors.

In Appendices A and B, we present graphical plots of the top-performing models selected based on the AIC: FIGARCH and EGARCH. Both models demonstrate strong performance in-sample and out-of-sample, effectively capturing the dynamics of cryptocurrency volatility. However, visual inspection reveals a tendency for both specifications to slightly underestimate extreme volatility movements. To account for this, practitioners may consider adopting a conservative approach, such as using a two-standard-deviation band around forecasts for risk management purposes.

A notable observation arises in the case of XRP, where the EGARCH model produces an unusually large spike in predicted volatility, exceeding the realised volatility during a single extreme event. This contrasts with the general trend of modest underestimation. In contrast, the FIGARCH model exhibits superior alignment with realised volatility across most periods, particularly for Ripple, suggesting a more robust capture of persistence and asymmetric effects.

Table 7. EGARCH (1,1) FIGARCH(1,d,1) Estimation Results with ARMA(1,0) Mean and GED Distribution.

Cryptocurrency	Model	Dataset	RMSE	MAE
BTC	EGARCH	Training	0.03066831	0.02385267
BTC	EGARCH	Test	0.02119651	0.01812959
BTC	FIGARCH	Training	0.03085954	0.02399308
BTC	FIGARCH	Test	0.02062238	0.01752228
ETH	EGARCH	Training	0.03641101	0.03077391
ETH	EGARCH	Test	0.02507182	0.02231952
ETH	FIGARCH	Training	0.03825686	0.02980853
ETH	FIGARCH	Test	0.02501502	0.02084927
XRP	EGARCH	Training	0.04976176	0.03450914
XRP	EGARCH	Test	0.08775667	0.03520921
BNB	EGARCH	Training	0.04347386	0.03125709
BNB	EGARCH	Test	0.02287451	0.01915384
BNB	FIGARCH	Training	0.04364836	0.03088031
BNB	FIGARCH	Test	0.02126344	0.01735932

Source: Created by Authors.

5. Discussion

The findings of this study provide critical insights into the efficacy of advanced GARCH specifications for modelling cryptocurrency volatility, addressing the research question of whether models incorporating asymmetry, regime-switching, and long-memory effects outperform traditional GARCH frameworks. Our results align with and extend existing literature while revealing nuanced dynamics unique to cryptocurrency markets.

A key contribution of our analysis is the systematic demonstration that regime-switching (MSGARCH), long-memory (FIGARCH), and asymmetric (EGARCH) models consistently outperform symmetric short-memory alternatives when modelling BTC, ETH, BNB, and XRP. This pattern parallels the findings of Naimy et al. [40], who also show that more advanced asymmetric GARCH-type models outperform simpler specifications for several cryptocurrencies, confirming that the volatility process of digital assets is inherently nonlinear. However, unlike their heterogeneous results—where different GARCH variants dominate for different coins—our study identifies MSGARCH as superior across all assets in-sample, emphasising the centrality of regime shifts in cryptocurrency markets.

The strong long-memory estimates for BTC and ETH are fully consistent with the evidence presented by Benzid & Saâdaoui [41] for the EUR/USD exchange rate: in both contexts, FIGARCH and HYGARCH capture persistent volatility better than short-memory models. In our case, FIGARCH dominates primarily for BTC and ETH, suggesting that long-memory is more pronounced in large-cap cryptocurrencies than in XRP and BNB. This interpretation is further strengthened by Mighri & Jaziri's [42] demonstration that long-memory and fat-tailed distributions materially improve VaR and ES forecasts—precisely the distributional improvements observed when using skewed Student's t or GED innovations in our study.

Our findings also align with Mensi et al. [43], who show that once structural breaks are accounted for, long-memory and volatility persistence decrease but remain economically meaningful. Their result supports our use of FIGARCH under structural regimes and confirms that the superior performance of long-memory models in BTC and ETH is not an artefact of unmodelled structural breaks. Because our data span multiple boom–bust cycles, this strengthens the conclusion that long-memory features are intrinsic rather than episodic.

The importance of regime-switching is further reinforced by Faruq et al. [44], who also find that MS-GARCH outperforms traditional GARCH models for the top-10 cryptocurrencies. Their evidence mirrors our in-sample dominance of MSGARCH and supports the interpretation that cryptocurrency volatility is better described by latent state transitions than by a single continuous volatility process. Our finding that MSGARCH converges poorly for BNB is consistent with their observation that model performance depends heavily on asset-specific features, adding nuance to the claim that regime-switching universally outperforms simpler models.

Recent literature on extreme market conditions provides further context. Pečiulis & Vasiliauskaitė [45] demonstrate that during crisis states, MS-GARCH and CS-GARCH outperform other specifications, corroborating our conclusion that the regime-switching mechanism captures tail-dependent, high-volatility periods better than symmetric models. Their finding that volatility in cryptocurrencies is largely endogenous and decoupled from global financial stress complements our own results showing that skewed distributions are required for accurate modelling even when classical financial stress indicators are absent.

Our results also intersect with the emerging literature on volatility spillovers. Korkusuz [46] shows that realized volatility spillovers are high and that ETH—not BTC—is the dominant transmitter. This supports our empirical observation that ETH exhibits stronger persistence (β coefficients closer to 1), which may reflect Ethereum's growing systemic role in DeFi markets. ETH's long-memory characteristics in our FIGARCH estimates are therefore economically aligned with its increasing network centrality.

At the same time, the broader literature underscores limitations in relying exclusively on GARCH models. Sun & Kristoufek [47] find that range-based estimators outperform GARCH-family models when high-frequency data serve as the benchmark for "true volatility." Their results suggest that part of the slight underestimation of extreme volatility we observe—

particularly in EGARCH for XRP—may arise from the inherent limitations of daily-frequency GARCH models rather than from model misspecification. Similarly, Queiroz & David [48] show that Realized-GARCH outperforms standard GARCH variants in out-of-sample forecasting when high-frequency information is included, reinforcing that the absence of intraday features in our dataset limits predictive precision.

The behavioural dimension of cryptocurrency markets also contributes to volatility characteristics we model. Gao et al. [49] find that bullish Twitter sentiment increases BTC return volatility at hourly frequencies. This aligns with our finding that conditional variance reacts strongly to recent shocks (high $\alpha+\gamma$ effects), and suggests that part of this responsiveness may be driven by sentiment-induced microstructure noise—an effect amplified in EGARCH and GJR-GARCH models but missed by symmetric GARCH. Because our dataset uses daily data, the inability to incorporate sentiment-driven intraday dynamics is an inherent limitation.

Furthermore, Ciarko et al. [50] highlight that cryptocurrencies' technological foundations, decentralisation, and security architecture create both structural opportunities and risks. Their emphasis on high volatility as an inherent weakness of cryptocurrencies supports the necessity of heavy-tailed error distributions in our study, while their identification of cybersecurity and infrastructure risks underscores why regime-switching models—capable of capturing abrupt spikes—perform best.

6. Limitations

First, computational challenges constitute a core limitation. FIGARCH and HYGARCH exhibited low convergence rates (49.8% and 37.5%), which is consistent with the mathematical complexity discussed by Shams et al. [51], who show that fractional-order methods require advanced iterative schemes to maintain numerical stability. Our convergence difficulties therefore reflect fundamental computational constraints rather than implementation deficiencies, constraining the practical scalability of long-memory volatility models. MSGARCH also demonstrated substantial convergence failures for BNB, in line with the asset-specific fragility documented by Pečiulis & Vasiliauskaitė [45]. These low convergence rates also influence the interpretation of comparative results. Although the successfully estimated specifications are valid, they represent only the subset of the parameter space where numerical optimisation is stable. Consequently, FIGARCH and HYGARCH may underrepresent theoretically relevant regions that are difficult to estimate but potentially informative. The comparative findings should therefore be viewed as reflecting the practically estimable behaviour of long-memory models rather than their full theoretical performance—an important distinction when assessing their role alongside MSGARCH and EGARCH.

Second, the risk of overfitting is non-trivial. While advanced models outperform simple GARCH in-sample, Sharma & Vipul [52] caution that added complexity does not necessarily translate into superior out-of-sample accuracy. Our results echo this: although MSGARCH dominates in-sample, FIGARCH and EGARCH outperform it in test sets, suggesting that regime-switching may overfit in periods lacking clear structural shifts.

Third, data frequency constraints limit generalisability. As Sun & Kristoufek [47], and Blasis [53] show, daily-frequency GARCH models miss intraday microstructure and realized volatility patterns, which affects accuracy for assets like XRP where intraday jumps are more common. Our slightly underestimated extreme volatility forecasts reflect this structural limitation.

Fourth, the generalisability of our results is limited. The dominance of long-memory models for BTC and ETH aligns with Benzid & Saâdaoui [41], and Mighri & Jaziri [42], but these patterns do not necessarily extend to smaller or newer digital assets. Income coins, DeFi tokens, or assets with limited liquidity may exhibit regime-switching or microstructure-driven volatility dynamics more consistent with WISMC or high-frequency frameworks [54].

Although hybrid or composite volatility models—such as Realized-GARCH, MS-FIGARCH, or FIAPARCH variants—could in principle capture additional layers of market behaviour, incorporating them into this study was not feasible without compromising the integrity of the comparison. Evaluating all possible hybrids across four assets would have expanded the estimation space far beyond the 6,720 specifications already computed and introduced significant convergence instability, as observed in the long-memory and regime-switching models included here. Recent evidence [42; 53] confirms that hybrid models often improve performance, but they also require high-frequency data or specialised optimisation frameworks, both outside the scope of this paper. Consequently, while our findings generalise well to large-cap cryptocurrencies with similar liquidity and volatility structures, they should be applied cautiously to smaller or structurally distinct assets, especially those exhibiting stronger microstructure noise, thinner order books, or instability in their volatility regime dynamics.

Finally, cryptocurrencies' interconnectedness implies that model performance may evolve as market structure changes. As Korkusuz [46] shows, ETH's rising influence in spillover networks indicates that volatility transmission mechanisms are dynamic; thus, models calibrated on historical dependencies may not generalise to future market states where the hierarchy of volatility transmitters shifts.

7. Conclusions

This study evaluated advanced GARCH specifications for forecasting the volatility of BTC, ETH, XRP, and BNB. The results show that conventional GARCH models are inadequate for cryptocurrencies, requiring specialised extensions. In-sample analysis revealed that MSGARCH and EGARCH outperformed symmetric and short-memory models, confirming the importance of regime-switching and leverage effects. MSGARCH was dominant, while EGARCH captured asymmetry effectively. FIGARCH showed superior fit for BTC and ETH, validating long-memory persistence. Skewed Student's t and GED distributions improved model fit.

Out-of-sample testing focused on EGARCH and FIGARCH. FIGARCH showed better accuracy for BTC and BNB, while EGARCH was more volatile, particularly for XRP. Both models slightly underestimated extreme volatility, suggesting conservative measures like two-standard-deviation bands for risk management.

These findings help refine hedging strategies and VaR estimations for traders and risk managers, while providing regulators with insights into market stability. Methodologically, this study sets a benchmark for volatility modelling in non-stationary environments.

Future research could explore hybrid models combining regime-switching and long-memory features or incorporate high-frequency data. Machine learning techniques alongside GARCH frameworks may further enhance predictions. Overall, advanced GARCH specifications are essential for robust cryptocurrency volatility forecasting and risk management.

Author Contributions: Conceptualisation, Methodology, Visualisation, Software, Validation, Formal analysis, Investigation, Data curation, Writing—original draft, T.P.; Writing—review and editing, T.P., and A.V.; Supervision, A.V. All authors have reviewed and approved the final version of the manuscript for publication.

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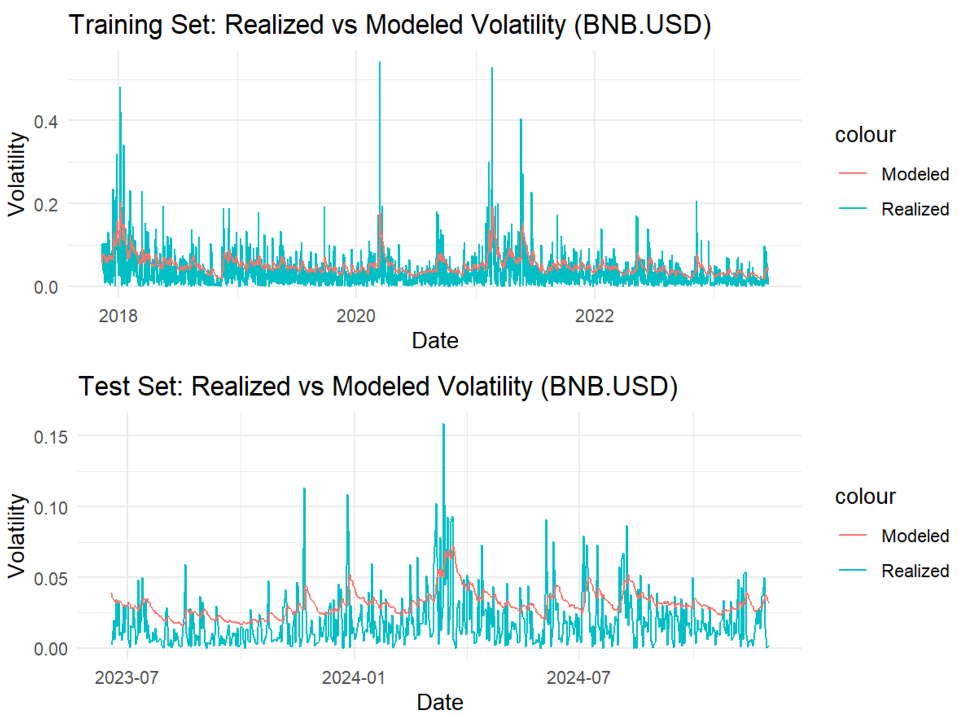
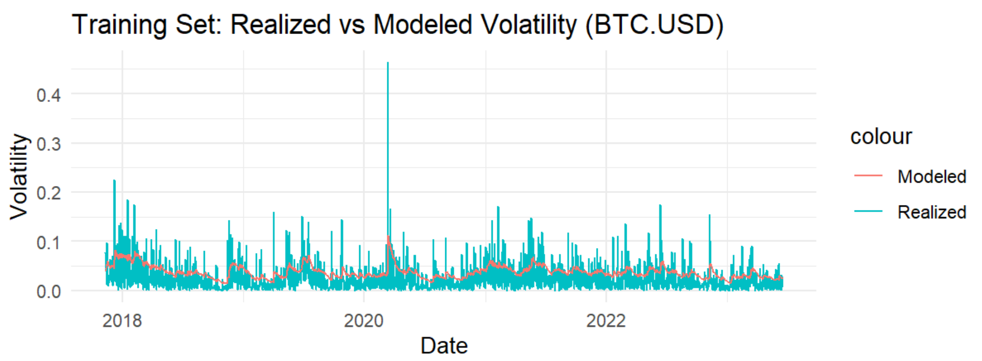
Statement on Data Availability: Data and code are available upon request via email: tomas.peciulis@vilniustech.lt

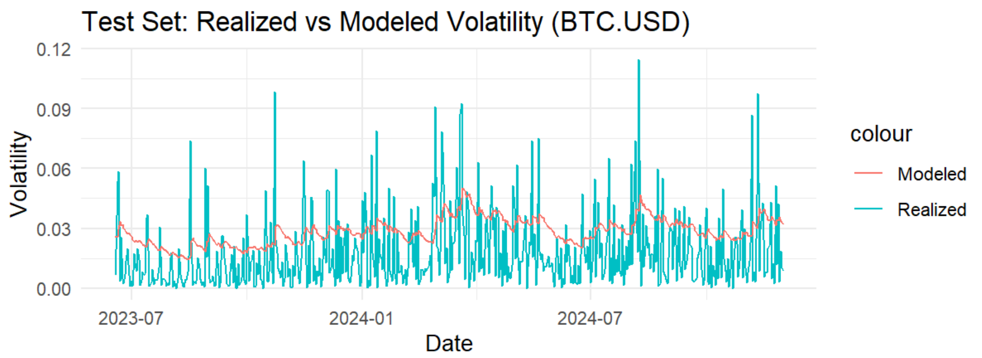
Conflicts of Interest: The authors declare no conflict of interest.

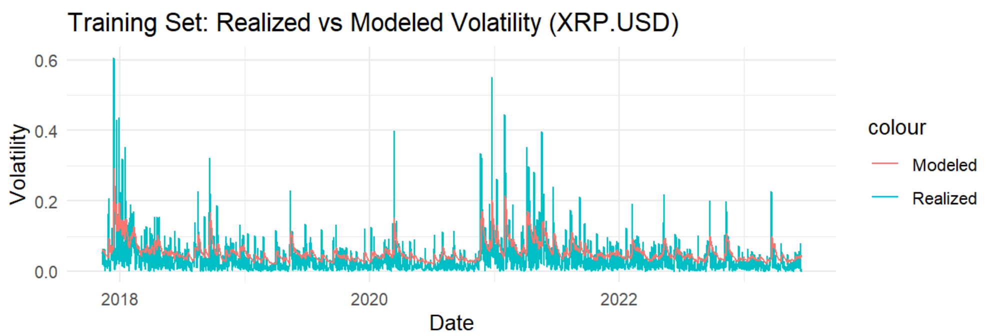
Appendix A.

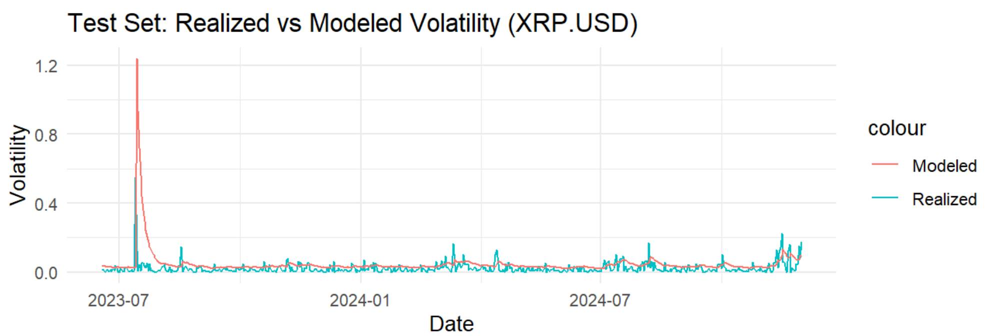
Comparison of ARMA (1,0) - EGARC (1, 1) with GED error distributions

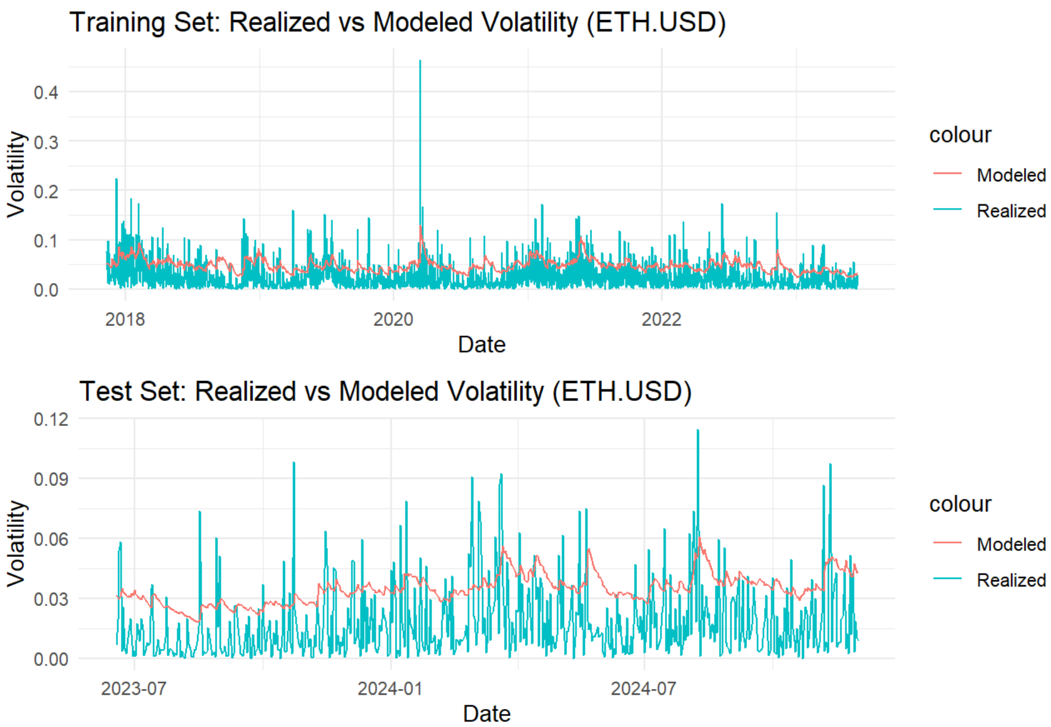



Training Set: Realized vs Modeled Volatility (BTC.USD)
Volatility
0.4
0.3
0.2
0.1
0.0
2018
2020
2022
Date
colour
Modeled
Realized


Test Set: Realized vs Modeled Volatility (BTC.USD)
Volatility
0.12
0.09
0.06
0.03
0.00
2023-07
2024-01
2024-07
Date
colour
Modeled
Realized

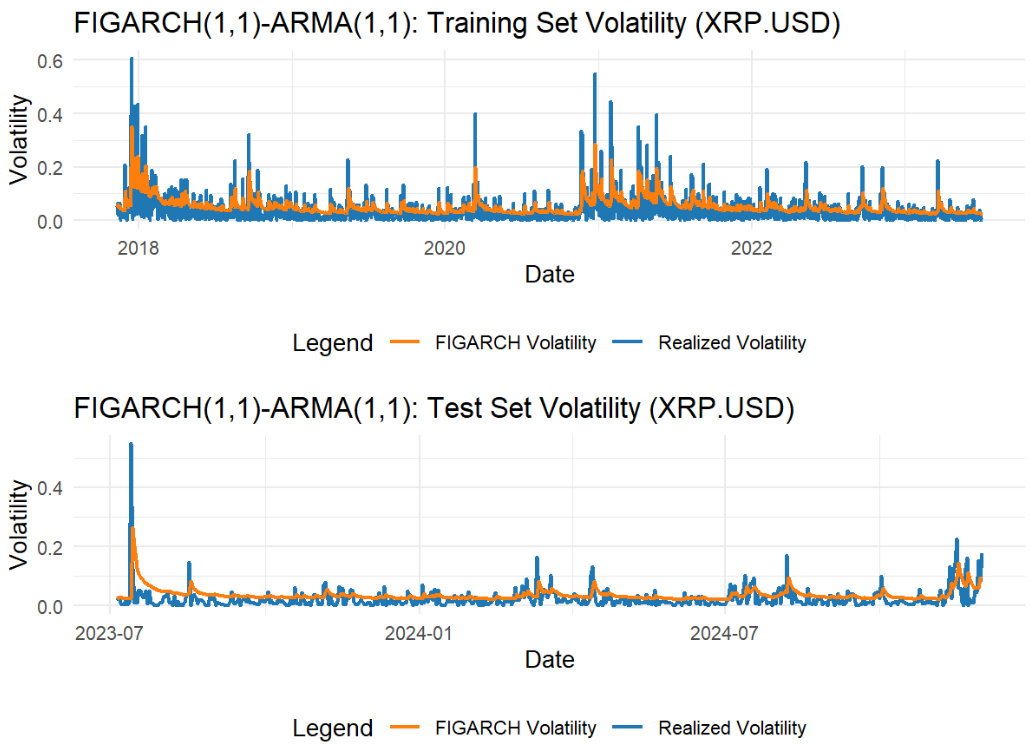

Training Set: Realized vs Modeled Volatility (XRP.USD)
Volatility
0.6
0.4
0.2
0.0
2018
2020
2022
Date
colour
Modeled
Realized

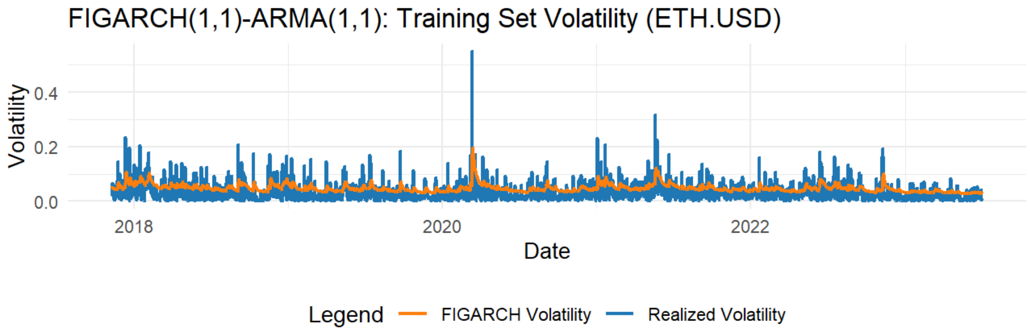

Test Set: Realized vs Modeled Volatility (XRP.USD)
Volatility
1.2
0.8
0.4
0.0
2023-07
2024-01
2024-07
Date
colour
Modeled
Realized

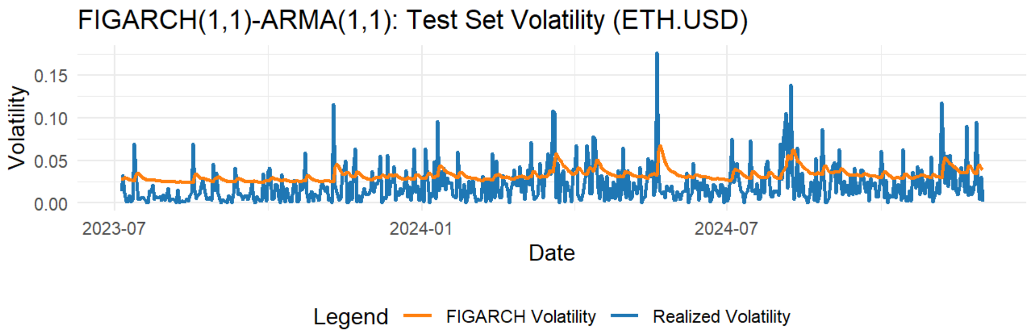


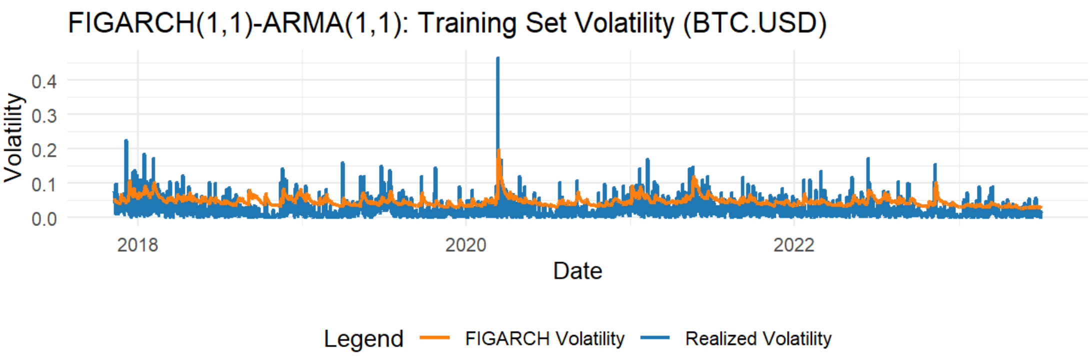
Appendix B.

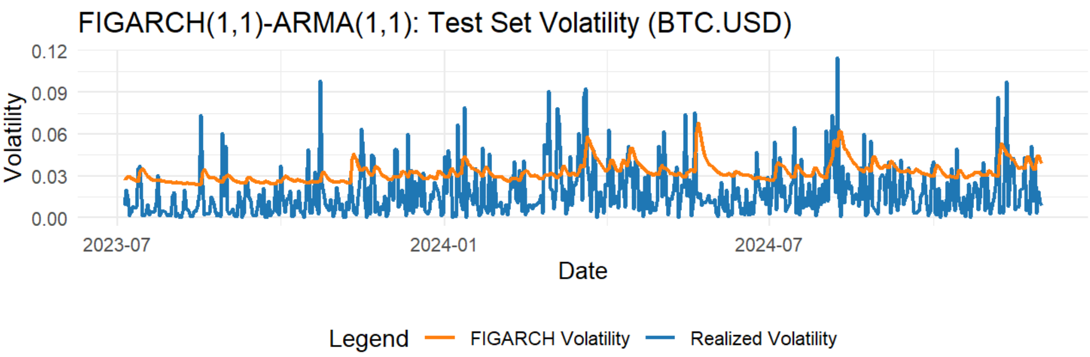
Comparison of ARMA (1,0) – FIARC (1, d, 1) with GED error distributions

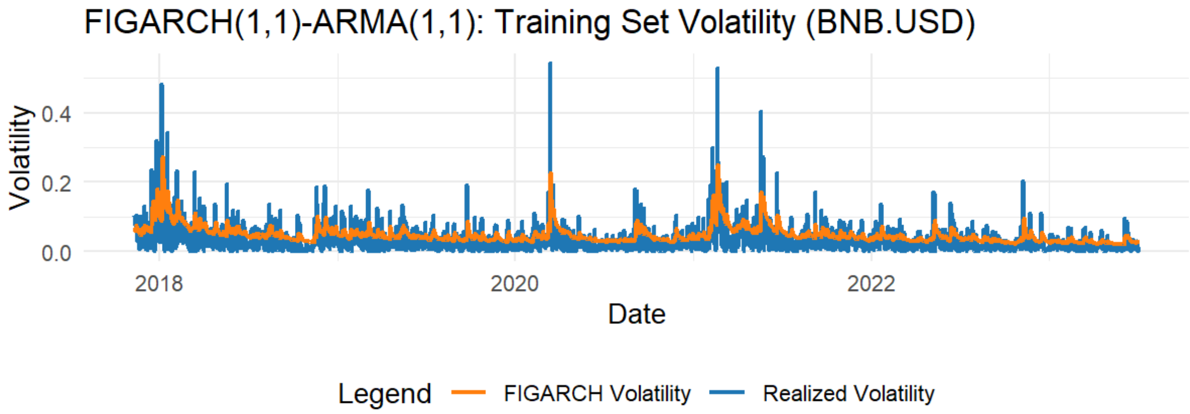


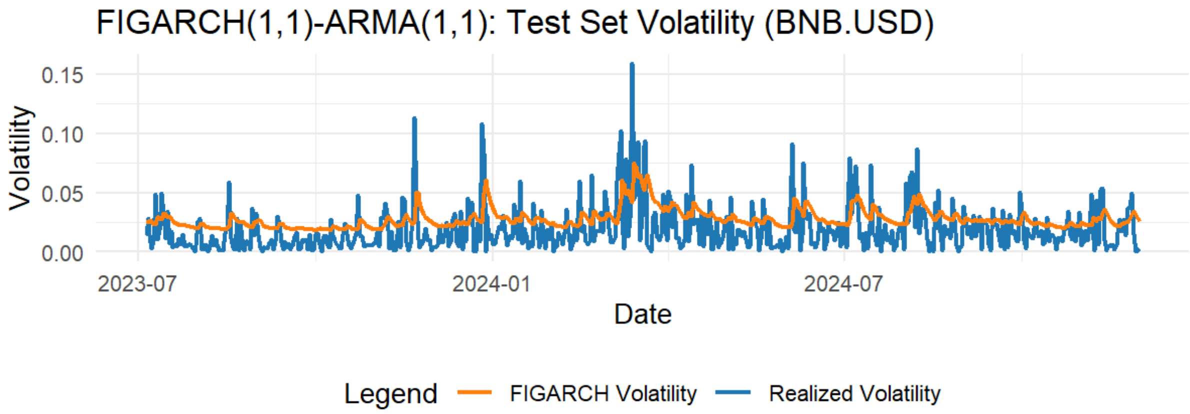

FIGARCH(1,1)-ARMA(1,1): Training Set Volatility (ETH.USD)
Volatility
0.4
0.2
0.0
2018
2020
2022
Date
Legend
FIGARCH Volatility
Realized Volatility


FIGARCH(1,1)-ARMA(1,1): Test Set Volatility (ETH.USD)
Volatility
0.15
0.10
0.05
0.00
2023-07
2024-01
2024-07
Date
Legend
FIGARCH Volatility
Realized Volatility


FIGARCH(1,1)-ARMA(1,1): Training Set Volatility (BTC.USD)
Volatility
0.4
0.3
0.2
0.1
0.0
2018
2020
2022
Date
Legend
FIGARCH Volatility
Realized Volatility


FIGARCH(1,1)-ARMA(1,1): Test Set Volatility (BTC.USD)
Volatility
0.12
0.09
0.06
0.03
0.00
2023-07
2024-01
2024-07
Date
Legend
FIGARCH Volatility
Realized Volatility





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